

# Design Of Discrete Time PID Control Of Aircraft Landing Gear

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**Abstract**— Different landing scenarios and runway unevenness have immediate effects on landing gear system performance. This decreases the pilot's ability to control the aircraft. Consequently, their vibrations result in structural fatigue. The discrete-time PID controller is used because of its simplicity of design and implementation. In this paper, the control strategy is carried out by a set of desired vertical displacement values for the model of a semi-active suspension system, and the error along the time intervals is then determined. The performance of a controlled landing gear under varying sink speeds is evaluated to use a passive suspension system. The results of a MATLAB simulation reveal that the discrete-time PID controller significantly improves the vertical vibration of the fuselage compared to the passive system, hence enhancing the landing quality and fatigue life of the structure under different operational conditions.

**Keywords**— PID control, MR damper, Aircraft landing gear, Oleo-pneumatic shock absorber.

## I. INTRODUCTION

The landing gears play a crucial function during touchdown and taxi as an intermediary component between the fuselage and the runway. The landing gear might rub the fuselage during taxiing due to road disturbances. The effects of aircraft vibrations vary with climatic conditions, landing gear construction, and disturbances. When moving on a rough runway, aircraft are subjected to significant vibrations. This creates significant vibrations on the fuselage, reduces passenger comfort, and may result in a catastrophic disaster. As a result, the landing gears must absorb the vertical kinetic energy [1].

Semi-active system control has theoretical and practical study areas. This depends on a number of parameters; semi-active systems are just as reliable as passive systems, and they need less energy than active ones. The magnetorheological (MR) damper includes controlled fluid. Vibrations are reduced with MR dampers by absorbing energy. In this technique, system stability will be preserved [2-5].

Vibration analysis utilizes many mathematical models of vehicle and aircraft systems. For comparing the semi-active system with passive and active systems, Margolis used a bond graph approach and demonstrated that optimum control techniques may be applied [6]. Omer and Rahmi [7] have created a semi-active suspension system for a 6 DOF vehicle model in order to enhance ride comfort, road holding, and

rattling speed. Sims and Stanway [8] have developed a semi-active suspension system for a quarter-sized vehicle that exhibits the better performance of semi-active suspensions over passive suspensions. Kruger constructed a semi-active landing gear employing three unique control strategies, and he compared the passive, active, and semi-active situations for a multi-body aircraft model [9].

In this study, MR damper landing gears are used to minimize aircraft landing and taxiing vibrations. The voltage of the MR damper is determined using a discrete-time PID controller.

In the last several decades, analog controls have often been replaced with digital controllers, which are characterized by having inputs and outputs that are set at discrete time periods. Digital controllers consist of digital circuits, digital computers, and microprocessors. It is recommended that a discrete-time PID controller be used instead of a continuous-time PID controller due to the many benefits that a discrete-time PID controller offers in comparison to a continuous-time PID controller. These benefits are demonstrated by the following: The most popular controller is the discrete-time closed-loop PID controller, whose response time is faster and whose rise time is shorter than that of the continuous-time PID controller [10]. The construction of the PID controller can be described by the block diagram shown in Figure 1.

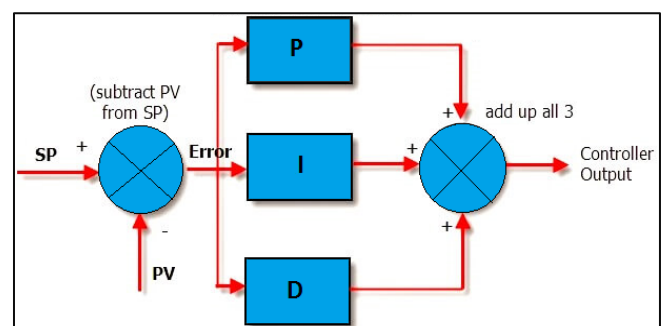


Fig. 1. Example Structure of PID controller.

## II. MATHMATECHAL MODEL

## A. Passive Main Landing Gear (MLG)

Relative to pneumatic, liquid and coil spring shock absorbers, oleo-pneumatic ones are the most effective. Which is why they are used in the majority of landing gears today. In addition to this, their ability to dissipate energy is unparalleled. In practical use, the efficiency of oleo-pneumatic dampers ranges from 80% to 90% [1]. Oil is contained in the top chamber of an oleo-pneumatic suspension system when the strut is compressed. This makes it possible to create a shock absorber that is more effective. The quantity of oil is adequate to provide the full stroke for which the damper is intended. The load for the necessary volumetric compensation that is needed during the stroke is carried by the amount of nitrogen  $N_2$ , that is squeezed above the oil level in the upper chamber. Figure 2 is a simplified schematic diagram of a strut. When the airplane is on the ground, the compressed nitrogen works as a spring to support its weight. Hydraulic fluid and compressed gas are contained in the upper chamber, which is shown by the light grey region in the Figure 2. The orifice plate acts as a separator between the upper and lower chambers[11].

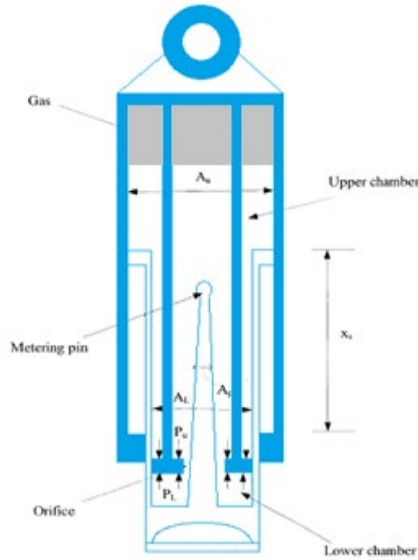


Fig. 2. A schematic diagram of a passive shock strut [11].

Figure 2 also shows the dynamic model of a single oleo-pneumatic damper. This passive aircraft landing gear mathematical model will be derived from a previous study [11-12].

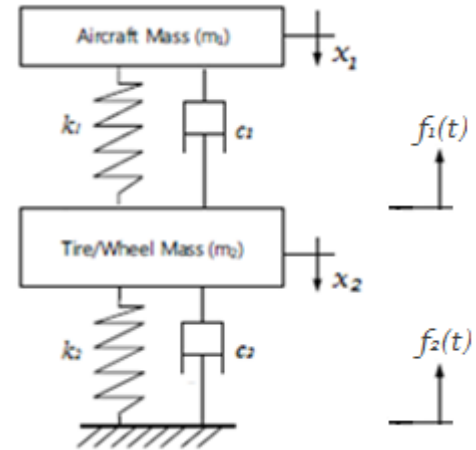


Fig. 3. Schematic model of the landing gear [11].

The equations of motion for the oleo-pneumatic system are represented as follows [11]:

$$m_1 \ddot{x}_1 + c_1(\dot{x}_1 - \dot{x}_2) + k_1(x_1 - x_2) = f_1(t) \quad (1)$$

$$m_2 \ddot{x}_2 + c_1(\dot{x}_2 - \dot{x}_1) + k_1(x_2 - x_1) + c_2 \dot{x}_2 + k_2 x_2 = f_2(t) \quad (2)$$

The system's previous equations of motion can be rearranged and written in matrix form as

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F\} \quad (3)$$

Where:  $\{x\}$  is the vector representing the displacement, The mass matrix's symbol is  $[M]$ , the matrix of damping coefficients is  $[C]$ ,  $[K]$  is the matrix of stiffness coefficients, and  $\{F\}$  is the force vector, respectively. Then substitute (1) and (2) for (3):

$$\{x\} = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad (4)$$

$$\{F\} = \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix} \quad (5)$$

$$[M] = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \quad (6)$$

$$[C] = \begin{bmatrix} c_1 & -c_1 \\ -c_1 & c_1 + c_2 \end{bmatrix} \quad (7)$$

$$[K] = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 + k_2 \end{bmatrix} \quad (8)$$

Equations (1) and (2) may be expressed as first-order differential equations in matrix form as follows:

$$\frac{d}{dx} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0_{2 \times 2} & I_{2 \times 2} \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0_{2 \times 2} \\ M^{-1} \end{bmatrix} \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix} \quad (9)$$

And by substitute the equation (5), equation (6), equation (7) and equation (8) in equation (9), the simulation can be carried out to obtain the time response of the masses displacement as will be seen in the simulation results.

### B. Semi-Active Main Landing Gear (MLG)

Figure 4 shows the structural and dynamic models of a single landing gear with an MR damper. Through small orifices in the piston head, MR fluid flows from one chamber to the other as the piston moves. Furthermore, the pressure difference between the chambers produces a viscous damping force. In addition to this traditional shock absorber configuration, the orifices are enclosed by two coils whose magnetic fields are controlled by an applied current. Micron-sized particles in the MR fluid flow along the magnetic field when an electric current is applied to the coils. This mechanism causes the MR fluid to transform into a viscoelastic solid in milliseconds, resulting in an increase in pressure differential and damping force. Thus, the damping force can be modified by adjusting the flow of electricity of the damper [13].

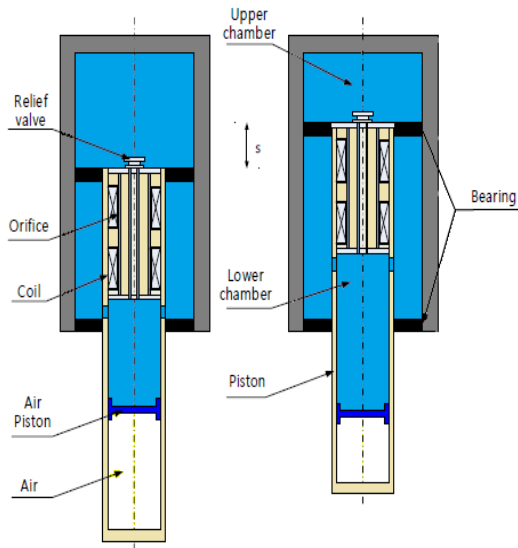


Fig. 4. Sketch of a shock strut equipped with a magneto-rheological (MR)[13].

Implementing semi-active dampers into an aircraft's shock absorber would provide a variable rate of energy dissipation. By adjusting the damping force within an acceptable range, it is also feasible to avoid the transmission of excess vibrations to the fuselage of the aircraft, particularly during landing. A smaller damping force, easily controllable by reducing the current, is adequate for other aircraft operating phases such as taxiing [14].

In this research, a two-degrees-of-freedom (2DOF) plane model was used in order to evaluate the dynamic response produced by the aircraft in the process of the landing contact. It is expected that the MR damper will take the place of the conventional damper in each of the three landing gears of an aircraft that uses tricycle landing gear. This model is shown in Figure 5.

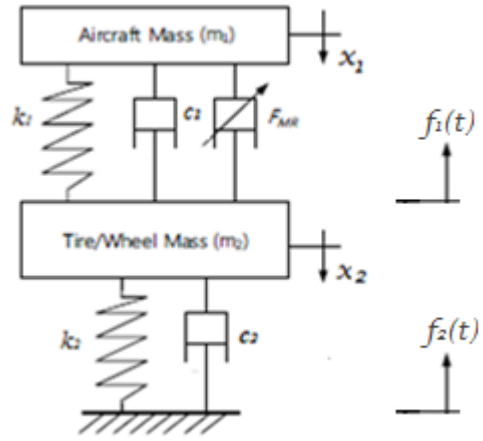


Fig. 5. The dynamic model of a single chassis with MR damper.

The following dynamic equilibrium equations for the fuselage and semi-active shock absorber system are obtained using Newton's law of motion and the system model [13]:

$$m_1 \ddot{x}_1 + c_1(\dot{x}_1 - \dot{x}_2) + k_1(x_1 - x_2) = f_1(t) + F_{MR} \quad (10)$$

$$m_2 \ddot{x}_2 + c_1(\dot{x}_2 - \dot{x}_1) + k_1(x_2 - x_1) + c_2 \dot{x}_2 + k_2 x_2 = f_2(t) - F_{MR} \quad (11)$$

### III. CONTROL DESIGN

The aircraft must be capable of landing in a variety of situations. Aircraft weight and sinking speed are the two main factors determining landing conditions. To design a control system effectively, it is necessary to correctly measure these variables. Although it is feasible to determine the sinking speed of an airplane using a ground sensor or accelerometer, it is hard to estimate the aircraft's mass with great precision since it relies on the fuel mass and residual payload (which may be neglected)[13].

The targets of the proposed controller are as follows:

- The controller improves the performance of the landing gear.
- The controller is capable of adapting to various landing scenarios.
- The controller can deal with unmodeled uncertainties and disturbances.

#### A. Landing Gear System Concept

Figure 6 shows the concept of a landing gear system with an MR damper. To reduce weight and equipment costs, the system consists of a single position sensor. After receiving the input signal generated by the sensor, the control technique calculates the necessary electric current, which is then supplied to the MR damper to provide the damping force. Designing a discrete-time PID to increase the performance of the landing gear is a crucial part of the system. After receiving the signal, the position sensor alters the potential difference between the two locations of the damper, which serves as the driving force for the electric current flow. The transformation of the current into a magnetic field raises the fluid's shear stress and provides the required damping force.

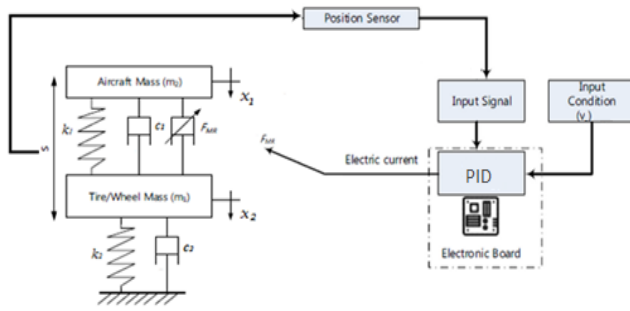


Fig. 6. Concept of a landing gear system.

### B. Discrete-time PID controller

A transformed digital signal must first be sampled before it can be converted from continuous time to discrete time. This permits the use of discrete intervals in the time domain to solve the difference equations and transform the signal from continuous to discrete time. Once the discrete-time PID controller has been applied to the signal, a plant needs an analog-to-digital converter to work [15].

The following is a representation of a PID controller:

$$u(t) = K_p[e(t) + \frac{1}{T_i} \int_0^t e(t) dt + T_d \frac{de(t)}{dt}] \quad (12)$$

For the PID controller to be implemented on a digital computer, the equation (12) must be converted from a continuous representation to a discrete one. There are numerous methods to achieve this, but the simplest is to employ the trapezoidal approximation for the integration and the backwards differences approach for the derivation, as shown below.

$$\int_0^t e(t) dt \approx \sum_{k=1}^n T e(Kt) \quad (13)$$

$$\frac{de(t)}{dt} \approx \frac{e(nT) - e(nT-T)}{T} \quad (14)$$

Putting these two approximations (13, 14) to equation (12) provides the following equations:

$$u(nT) = K_p[e(nT) + \frac{1}{T_i} \sum_{k=1}^n T e(KT) + T_d \frac{e(nT) - e(nT-T)}{T}] \quad (15)$$

$$u(nT - T) = K_p[e(nT - T) + \frac{1}{T_i} \sum_{k=1}^{n-1} T e(KT) + T_d \frac{e(nT-T) - e(nT-2T)}{T}] \quad (16)$$

Subtracting these two equations, we obtain:

$$u_n = u_{n-1} + K_p[e_n - e_{n-1}] + \frac{K_p T}{T_i} e_n + \frac{K_p T_d}{T} [e_n - 2e_{n-1} + e_{n-2}] \quad (17)$$

$$u_n = u(nT) \quad (18)$$

$$u_{n-1} = u(nT - T) \quad (19)$$

$$e_n = x_{sp} - x_{pv} \quad (20)$$

Where:

$T$  is the sampling time,

$K_p, T_d, T_i$  are the proportional gain, derivative time gain and integral time gain respectively.

$x_{sp}$  is the desired set point.

$x_{pv}$  is the process value.

The controlled responses are obtained by multiplying the PID controller by the open systems  $x_1$  and  $x_2$ .

## IV. NUMERICAL SIMULATION

The computation of the response of the A6 Intruder aircraft to landing employs the various system properties that are tabulated in Table I. These computations are used to estimate how well the aircraft will respond [16].

TABLE I. LANDING GEAR PARAMETER.

Symbol	Quantity	Value	Unit
$m_1$	Partial mass of the aircraft	4928	$Kg$
$m_2$	Mass of the tire and the piston	148	$Kg$
$c_1$	Viscous damping coefficient	48.2~102.7	$KN.s/m$
$k_1$	Gas stiffness coefficient	64	$KN/m$
$c_2$	Damping coefficient of the tire	5	$KN.s/m$
$k_2$	Stiffness of the tire	1080	$KN/m$
$L_0$	Length of MR shock strut	0.38	$m$
$F_{MR}$	Control input (MR force)	0~30	$KN$

Sink speed at touchdown varies between landing operations; hence, two velocities were chosen. The initial conditions of the two-DOF system can be expressed as [16]:

Normal Landing:

$$\begin{aligned} x_1(0) &= x_2(0) = 0, \\ \dot{x}_1(0) &= \dot{x}_2(0) = 1.5 \text{ m/s} \end{aligned} \quad (21)$$

Hard landing:

$$\begin{aligned} x_1(0) &= x_2(0) = 0, \\ \dot{x}_1(0) &= \dot{x}_2(0) = 3.2 \text{ m/s} \end{aligned} \quad (22)$$

### A. Results And Discussion

The mathematical model for the considered system is simulated when the input sine wave with an amplitude of 100 mm and a frequency of 145.4 rad/s is applied and the given initial conditions of the system states are considered. Based on the results, MLG on passive and semi-active cases is analyzed. The results are given only for the bounce of the fuselage and the vertical displacements. Neither pitch nor roll motions are taken into consideration. Semi-active and passive cases are compared.

The results are shown in figures 7 to 10. It shows the response of the passive case at two velocities, simulating a hard landing and a normal landing. Similarly, the time responses of the compensates system with PID controller are shown respectively from figure.11 to figure. 14. Where the designed PID controller gains are selected arbitrarily ( $K_p = 6$ ,  $T_i = 1.4$ , and  $T_d = 2.8$ ) and no tuning methods were considered at any step in the simulation of the system.

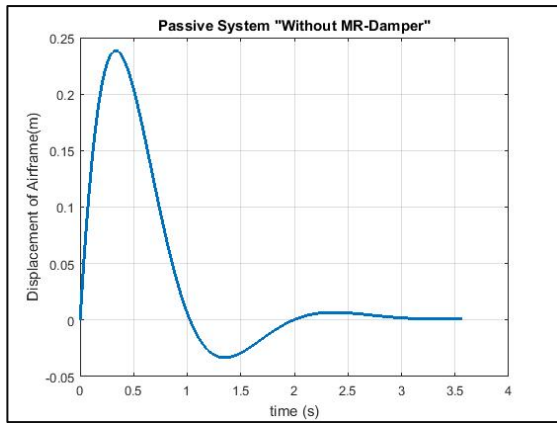


Fig. 7. Response of fuselage system under hard landing.

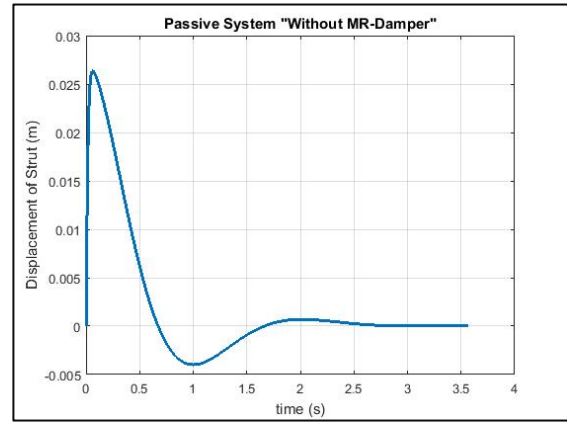


Fig. 10. Response of strut under normal landing.

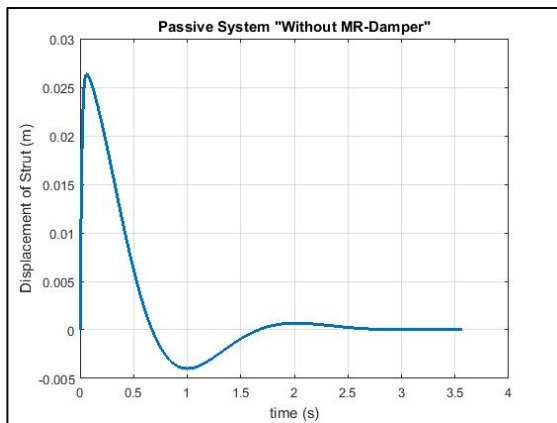


Fig. 8. Response of fuselage under normal landing.

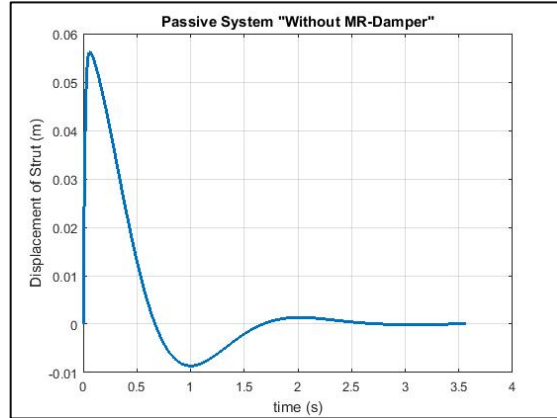


Fig. 9. Response of strut under hard landing.

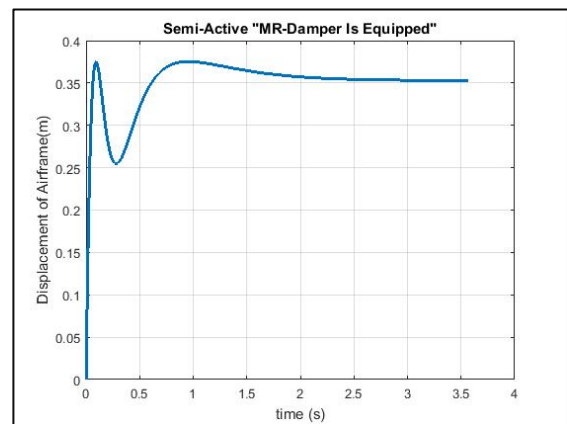


Fig. 11. Response of fuselage under hard landing.

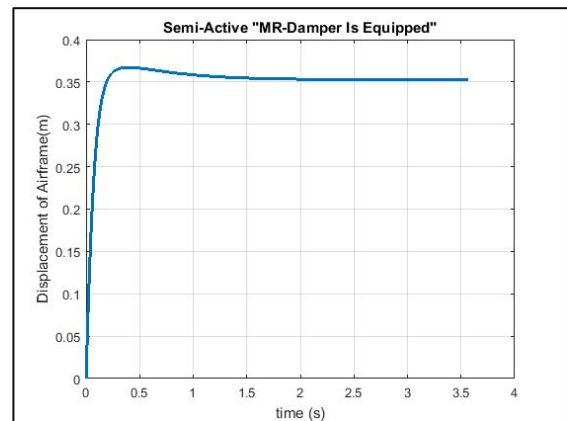


Fig. 12. response of fuselage under normal landing.

As can be observed, the displacement of the higher mass decreases slightly after  $t = 0.4$ s due to the orifice reducing effect. It should be noted that the displacements of the lower mass are within the allowed range of tire deflection, which, according to [16], does not exceed 0.09m. After that, a discrete-time PID controller is designed by inserting an MR damper into the landing system, where it is regarded as an external force within the range  $F_{MR} = 0 \sim 30$  kN that influences the system's response and controls its adjustment toward the desired response, which is given as the set point  $x_{1\text{desired}} = 0.35$ m.



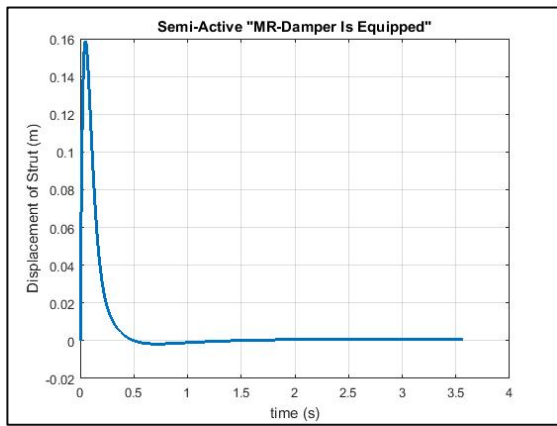


Fig. 13. Response of strut under hard landing.

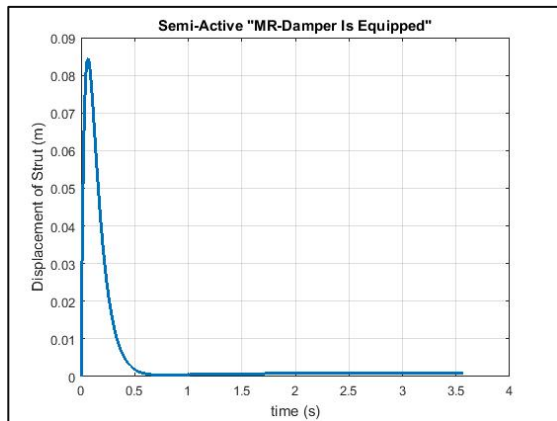


Fig. 14. Response of strut under normal landing.

TABLE II. PERFORMANCE SPECIFICATIONS OF HARD LANDING.

Case	Overshoot (m)	Rise time (s)	Settling time (s)
Semi-active	0.02	0.1	1.5
Passive	0.24	0.2	2.4

As can be seen, the displacement of the upper mass decreases significantly after  $t = 0.15$  s when the current is increased. Lower mass displacements are within the permitted range of tire deflection as tire force for a normal landing but not for a hard landing; nonetheless, other tire specifications may be used. Upper mass displacements are likewise within the allowed range since the fully expanded stroke of the shock strut was determined to be  $x_{1,max} = 0.38m$  based on the work of Daniels et al. [17]. where the set point for upper mass  $x_{1,ddesired} = 0.35m$  was chosen.

## V. CONCLUSIONS AND FUTURE WORK

This semi-active approach modifies the damping quality by varying the orifice size of the shock absorber subsystem in an oleo-pneumatic suspension system. Using a PID controller, the response of semi-active landing gear is determined. MATLAB-Script has been used to calculate the landing gear response of an aircraft with and without an MR damper. In addition, the operating conditions have altered, which is represented by the sinking speed, which ranges from normal landing to hard landing. Consequently, it can be deduced that

the performance of semi-active landing gear for PID with discrete intervals is superior to that of a passive system. However, tuning methods are required for the system to improve performance. Implementation of the discrete-time PID controller and optimum controllers such as genetic algorithm, ant colony, and particle swarm optimization are in the future scope of this study.

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